

SIMULTANEOUS HEAT AND MASS TRANSFER IN FILM ABSORPTION UNDER LAMINAR FLOW

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Abstract—This paper describes a theoretical analysis of the combined heat and mass transfer process taking place in the absorption of a gas or vapor into a laminar liquid film. The energy and diffusion equations are solved simultaneously to give the temperature and concentration variations at the liquid–gas interface and at the wall. Two cases of interest are considered: a constant temperature and an adiabatic wall. The Nusselt and Sherwood numbers are found to depend on the Peclet and Lewis numbers as well as on the equilibrium characteristics of the working materials.

NOMENCLATURE

<p>A_n, B_n, coefficients in equations (13a) and (13b);</p> <p>$a_{n,i}, b_{n,i}$, coefficients in equations (19) and (20);</p> <p>C, concentration of absorbate in solution [mol m^{-3} solution];</p> <p>C_e, equilibrium concentration of solution at temperature T_0 with vapor at pressure P_v [mol m^{-3} solution];</p> <p>C_i, interfacial concentration of absorbate in solution [mol m^{-3} solution];</p> <p>C_0, initial concentration of absorbate in solution [mol m^{-3} solution];</p> <p>C_1, C_2, constants in equation (11);</p> <p>c_p, specific heat of liquid [$\text{J kg}^{-1} \text{ }^\circ\text{C}^{-1}$];</p> <p>$D$, diffusion coefficient of absorbate (substance II) in solution [$\text{m}^2 \text{ s}^{-1}$];</p> <p>$F_n, G_n$, eigenfunctions in equations (13a) and (13b);</p> <p>\bar{H}_a, heat of absorption of substance II in solution [J mol^{-1}];</p> <p>\bar{H}_{II}, partial molal enthalpy of substance II at interface [J mol^{-1}];</p> <p>h_M, mass transfer coefficient from interface to bulk [m s^{-1}];</p> <p>h_T, heat transfer coefficient from interface to bulk [$\text{W m}^{-2} \text{ }^\circ\text{C}^{-1}$];</p> <p>$h'_T$, heat transfer coefficient from bulk to wall [$\text{W m}^{-2} \text{ }^\circ\text{C}^{-1}$];</p> <p>$\bar{h}_v$, enthalpy of vapor in contact with film [J mol^{-1}];</p> <p>k, thermal conductivity of liquid [$\text{W m}^{-1} \text{ }^\circ\text{C}^{-1}$];</p> <p>$Le$, Lewis number, D/α;</p> <p>n_i, mass flux of absorbate into absorbent at interface [$\text{mol m}^{-2} \text{ s}^{-1}$];</p> <p>$Nu, Nu'$, Nusselt numbers, $h_T \Delta/D$ and $h'_T \Delta/D$;</p> <p>Pe, Peclet number, $\bar{u} \Delta/\alpha$;</p> <p>P_v, vapor pressure of absorbate (substance II) in the gas phase [Pa];</p> <p>P_{v0}, vapor pressure of absorbate (substance II) in solution at concentration C_0 and temperature T_0 [Pa];</p> <p>q_w, normalized heat flux, $[\partial\theta/\partial\eta]_{\eta=0}$;</p> <p>$Sh$, Sherwood number, $h_M \Delta/D$;</p> <p>T, temperature of solution [$^\circ\text{C}$];</p>	<p>T_e, equilibrium temperature of solution at concentration C_0 with vapor at pressure P_v [$^\circ\text{C}$];</p> <p>T_i, interfacial temperature of solution [$^\circ\text{C}$];</p> <p>T_0, initial temperature of solution [$^\circ\text{C}$];</p> <p>u, flow velocity [m s^{-1}];</p> <p>\bar{u}, average flow velocity [m s^{-1}];</p> <p>v, normalized velocity, equation (6);</p> <p>x, coordinate in direction of flow [m];</p> <p>y, coordinate in direction perpendicular to flow [m].</p> <p>Greek symbols</p> <p>α, thermal diffusivity of liquid [$\text{m}^2 \text{ s}^{-1}$];</p> <p>$\alpha_n, \beta_n$, eigenvalues in equations (13a) and (13b);</p> <p>γ, normalized concentration, equation (6);</p> <p>$\gamma_i, \gamma_w, \bar{\gamma}$, normalized concentration at interface, wall and bulk, respectively;</p> <p>Δ, film thickness [m];</p> <p>ζ, normalized coordinate in direction of flow, equation (6);</p> <p>η, normalized coordinate perpendicular to flow, equation (6);</p> <p>η_1, $(1 - \eta)$, equation (27);</p> <p>θ, normalized temperature, equation (6);</p> <p>$\theta_i, \theta_w, \bar{\theta}$, normalized temperature at interface, wall and bulk, respectively;</p> <p>λ, normalized heat of absorption, equation (10);</p> <p>μ_i, normalized mass flux at interface, equation (10);</p> <p>ρ, density of liquid [kg m^{-3}].</p>
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1. INTRODUCTION

ABSORPTION of gases and vapors in liquids are encountered in numerous applications in the chemical technology. These processes normally involve simultaneous heat and mass transfer in the gas–liquid system. The heat of absorption gives rise to temperature gradients leading to the transfer of heat; the temperature influences the vapor pressure–concentration equilibrium between the two phases which in turn affects the exchange of mass.

The combined heat and mass transfer process does not lend itself easily to mathematical analysis. Many studies of absorption problems described in the literature have considered the heat and the mass transfer separately, neglecting the coupling between them. Fortunately, in many real cases the heat interaction is small and the process may be considered isothermal. In some processes, however, the effect of heat transfer is important and cannot be neglected. A typical example is when the absorbate is a vapor with high heat of absorption, such as water. Furthermore, there is growing interest in processes where mass transfer is initiated specifically to produce a temperature change. One such example, from which the present study originated, is in absorption heat pumps for heating and cooling. There, the heat transfer accompanying the mass transfer is of primary importance.

The gas-liquid contactors in which absorption takes place are typically spray, trayed, or packed towers. Of particular interest are systems involving falling liquid films, which have found wide application in modern equipment. A considerable number of studies have been performed on gas absorption in liquid films with different flow regimes, geometries, and boundary conditions. Chien and Ibele [1] gave a comprehensive survey on the hydrodynamics of falling films. Vyazovov [2] formulated, as early as 1940, a simple model for isothermal absorption in a falling film, which was shown by comparison with experimental results to provide rough estimates for the mass transfer coefficients. Improved and more elaborate models have been developed since. Olbrich and Wild [3] provided a solution to the diffusion equation in laminar flow for several falling film geometries. The solution, in the form of a series of eigenfunctions, includes ten eigenvalues and coefficients. Rotem and Neilson [4] added to the laminar solution the diffusion in the direction of flow, which turns out to be negligible for large enough Peclet numbers. Tamir and Taitel [5] extended the laminar flow solution to cases involving interfacial resistance. Chavan, Mechelkar and Karanth [6, 7] considered absorption in non-Newtonian liquids. Sandall and co-workers [8-10] studied turbulent flows. The common feature to all the above studies is their dealing with mass transfer only, under conditions where heat transfer has no effect.

Only recently has some work been published on combined heat and mass transfer in falling films. Yih and Seagrave [11] analyzed a laminar flow problem and studied the effect of a temperature gradient on the absorption process. Neglecting temperature variations in the direction of flow, they essentially assumed a linear temperature profile across the film thickness. The temperature variation in their model influenced the process through its effect on the physical properties of the liquid. Nakoryakov and Grigor'eva [12] used a similar approximate approach and assumed also a linear temperature profile across the film. However, in their model temperature variations in the direction of

flow were not neglected. Two later and improved models [13, 14] by the same authors calculated, rather than assumed, the actual shape of the temperature profile, which led to more accurate results. In ref. [13] an eigenfunction series solution is given for the coupled diffusion and energy equations with an impermeable, constant temperature wall and an equilibrium boundary condition at the liquid-vapor interface. In ref. [14] an analytic solution was obtained for the temperature and concentration variation near the entrance region. The main limitation of the models [13, 14] is their being based on the assumption of a uniform velocity profile in the film, whereas the actual velocity profile in laminar flow is parabolic. This assumption leads to a deviation of about 20% in the heat and mass transfer coefficients and to underprediction by about 40% of the distance required for boundary layer development. Also, the models [13, 14] are restricted to a constant temperature wall.

This paper presents an attempt to improve upon the models described earlier and eliminate some of their limitations. The model, for a falling film of absorbent solution in laminar flow, aims at the calculation of the heat and mass transfer coefficients for typical wall conditions and finding their dependence on the system's parameters.

2. MODEL AND EQUATIONS

The system analyzed in the present study is described schematically in Fig. 1. A film of liquid solution, composed of substances I (absorbent) and II (absorbate), flows down over an inclined plane. Substance I always remains in the liquid phase; substance II may be absorbed into the solution. The film is in contact with stagnant vapor of substance II at constant pressure P_v . At $x = 0$, the liquid solution is at a uniform temperature T_0 and composition C_0 (moles of II per unit volume of solution) corresponding to an equilibrium vapor pressure $P_{v,0}$ different from P_v . As a result of this difference, a mass transfer process takes place at the liquid-vapor interface. The substance absorbed at the interface diffuses into the film; the heat generated in the absorption results in a simultaneous

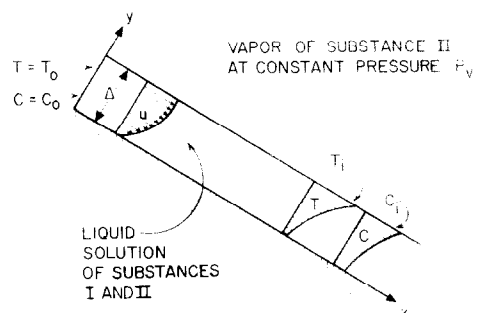


FIG. 1. Description of falling film of absorbent in contact with absorbate. Typical profiles of velocity, temperature, and concentration are shown.

heat transfer process. Two cases of practical interest are considered: in one, the wall is kept at a constant temperature T_0 ; in the other, the wall is adiabatic.

The flow of the liquid film is assumed in this study to be laminar, 1-dim., and fully developed throughout. No shear forces are exerted on the liquid by the vapor. The film thickness can be easily determined from the mass flow rate, density, viscosity, and angle of inclination [1]. The velocity profile, shown in Fig. 1, is parabolic and given by

$$u = \frac{3}{2}\bar{u}\left[2\left(\frac{y}{\Delta}\right) - \left(\frac{y}{\Delta}\right)^2\right] \quad (1)$$

where \bar{u} is the average flow velocity, equal to the mass flow rate per unit breadth, divided by the density and film thickness.

In formulating this model, the following assumptions have been made:

- (1) The liquid solution is Newtonian and its physical properties are constant and independent of temperature and concentration.
- (2) The mass of vapor absorbed per unit time is small compared to the mass flow rate of the liquid. Therefore, it is assumed that the latter is constant, and so are the film thickness and average flow velocity.
- (3) There is no heat transfer in the vapor phase.
- (4) There are no natural convection effects in the film due to temperature or concentration differences (this assumption is in fact a corollary of the first one).
- (5) Diffusion thermal effects are negligible.
- (6) Vapor pressure equilibrium exists between the vapor and liquid at the interface.

Under the above assumptions, the simultaneous heat and mass transfer in the system at steady-state is described by the energy and diffusion equations:

$$u \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial y^2}, \quad (2)$$

$$u \frac{\partial C}{\partial x} = D \frac{\partial^2 C}{\partial y^2} \quad (3)$$

where diffusion and heat conduction in the x -direction have been neglected with respect to those in the y -direction. The following boundary conditions apply:

$$T = T_0 \quad \text{and} \quad C = C_0 \quad \text{at} \quad x = 0, \quad (4a)$$

$$\left. \begin{array}{l} \frac{\partial C}{\partial y} = 0 \\ \left. \begin{array}{l} T = T_0 \quad \text{for constant temperature wall} \\ \partial T / \partial y = 0 \quad \text{for adiabatic wall} \end{array} \right\} \text{at } y = 0, \end{array} \right\} \quad (4b)$$

$$T = T_i \quad \text{and} \quad C = C_i \quad \text{at} \quad y = \Delta. \quad (4c)$$

Here T_i and C_i are the interfacial temperature and concentration, both unknown functions of x . They are related to each other and to the interfacial mass flux n_i ,

also unknown, by the following three conditions:

$$F(T_i, C_i) = P_v = \text{const.}, \quad (5a)$$

$$D(\partial C / \partial y) = n_i \quad \text{at} \quad y = \Delta, \quad (5b)$$

$$k(\partial T / \partial y) = n_i \bar{H}_a(T_i, C_i) \quad \text{at} \quad y = \Delta \quad (5c)$$

where \bar{H}_a is the heat of absorption (per mole of the vapor) in the liquid. Equation (5a) represents the condition of vapor pressure equilibrium at the interface; equations (5b) and (5c) describe the mass and heat fluxes, respectively, at the interface. The heat of absorption is defined as

$$\bar{H}_a = \bar{h}_{II} - \bar{H}_{II}(C_i, T_i) \quad (5d)$$

where \bar{h}_{II} is the enthalpy (per mole) of the vapor in contact with the film and \bar{H}_{II} is the partial molal enthalpy of substance II at the interface. \bar{H}_{II} is a function of the interfacial temperature and concentration whereas \bar{h}_{II} is independent of them. The definition (5d) is more rigorous than the one sometimes found in the literature, where \bar{H}_a is expressed in terms of the latent heat of vaporization/condensation of substance II at temperature T_i less the differential heat of dilution. This definition would be correct if the vapor was saturated at a temperature equal to that of the liquid interface. This is generally not the case, nor is it so in the present problem.

The typical shapes of the temperature and concentration profiles in the film are depicted in Fig. 1. Before proceeding with the solution, it would be useful to rewrite the equations in a dimensionless form. Let us define the new variables

$$\zeta = \frac{1}{Pe} \frac{x}{\Delta}; \quad \eta = \frac{y}{\Delta}, \quad (6a)$$

$$v = u/\bar{u} = \frac{3}{2}(2\eta - \eta^2), \quad (6b)$$

$$\theta = \frac{T - T_0}{T_e - T_0}; \quad \gamma = \frac{C - C_0}{C_e - C_0} \quad (6c)$$

where T_e is the equilibrium temperature of the solution at concentration C_0 with the vapor, and C_e is the concentration of the solution at temperature T_0 in equilibrium with the vapor. T_e and C_e both have a physical significance: T_e is the temperature which would be reached in the film if thermodynamic equilibrium could be achieved without change in concentration; C_e is the concentration that would be reached if thermodynamic equilibrium could be achieved without change in temperature. Both are limiting cases to what actually happens in the simultaneous heat and mass transfer process.

Equations (2) and (3) with the new dimensionless variables become

$$v \frac{\partial \theta}{\partial \zeta} = \frac{\partial^2 \theta}{\partial \eta^2}, \quad (7)$$

$$v \frac{\partial \gamma}{\partial \zeta} = Le \frac{\partial^2 \gamma}{\partial \eta^2} \quad (8)$$

where Le is the Lewis number. The boundary

conditions now have the dimensionless form

$$\theta = 0 \quad \text{and} \quad \gamma = 0 \quad \text{at} \quad \zeta = 0, \quad (9a)$$

$$\left. \begin{array}{l} \frac{\partial \gamma}{\partial \eta} = 0 \\ \left\{ \begin{array}{l} \theta = 0 \quad \text{for constant temperature wall} \\ \partial \theta / \partial \eta = 0 \quad \text{for adiabatic wall} \end{array} \right\} \end{array} \right\} \text{at } \eta = 0, \quad (9b)$$

$$\theta = \theta_i \quad \text{and} \quad \gamma = \gamma_i \quad \text{at} \quad \eta = 1 \quad (9c)$$

where θ_i and γ_i are the dimensionless interfacial temperature and concentration, both unknown functions of ζ , which are related to each other as follows:

$$f(\theta_i, \gamma_i) = 0 \quad (\text{equilibrium condition}), \quad (10a)$$

$$\frac{\partial \gamma}{\partial \eta} = \mu_i = \frac{n_i \Delta}{D(C_e - C_0)} \quad \text{at} \quad \eta = 1, \quad (10b)$$

$$\frac{\partial \theta}{\partial \eta} = \mu_i \lambda(\theta_i, \gamma_i) = \mu_i Le \frac{\bar{H}_a(C_e - C_0)}{\rho c_p (T_e - T_0)} \quad \text{at} \quad \eta = 1. \quad (10c)$$

Here μ_i is the dimensionless mass flux from the vapor to the film and λ is the dimensionless heat of absorption, which is a function of θ_i and γ_i .

The problem is now well defined mathematically in terms of the two second order differential equations (7) and (8) and the boundary conditions (9) for the unknown distributions of θ and γ with ζ and η . The boundary conditions are given in terms of two additional unknowns, θ_i and γ_i , which are determined along with μ_i from the three additional equations (10).

As has been stated earlier, the two cases for which the present model was developed (constant temperature and adiabatic wall) are of practical interest in actual working systems. The former simulates a process where the liquid is constantly cooled during absorption, such as in absorption chillers and heat pumps. The other is representative of a case where this process occurs without cooling, such as in many gas-liquid contactors. We have assumed here that the constant temperature wall is at a temperature T_0 equal to that of the entering solution. If this is not the case, the results will vary somewhat due to an additional pure heat transfer process between the wall and the film near the entrance region [14]. Also, the adiabatic wall may be considered as a particular case of the more general constant heat flux condition.

3. THE LINEAR ABSORBENT

In order to proceed with the solution, it is necessary to know the equilibrium relation between the temperature, composition, and vapor pressure of the specific liquid absorbent being used. This relation, expressed in a dimensionless form for the parameters at the interface, yields equation (10a). In addition, it is

necessary to express the dimensionless heat of absorption λ in terms of θ_i and γ_i for the given materials. Data on equilibrium properties have been compiled from experimental and theoretical studies and are available in the literature for many liquid-vapor combinations.

A universal relation between the temperature, concentration, and vapor pressure in equilibrium can be formulated which would fit a large number of absorbents within a limited range of the preceding parameters. This relation indicates a linear dependence between the temperature, the concentration, and the logarithm of the vapor pressure. A thermodynamic justification for this relation, limited to electrolytic solutions, has been given [15], and was based on the definition of the osmotic coefficient and the Clapeyron equation. However, similar behavior is exhibited by some other, non-electrolytic, absorbents. The validity of the linear relation was checked for two common absorbents, LiBr-H₂O and LiCl-H₂O, and found to be very good under the above limitations for a wide range of temperatures and concentrations.

The heat of absorption has been defined as the difference between the enthalpy of the vapor \bar{h}_v and the partial molal enthalpy of substance II in the liquid \bar{H}_{II} [equation (5d)]. \bar{h}_v , which is independent of the interfacial temperature and concentration, is often considerably larger than \bar{H}_{II} . This is so particularly with vapor of low molecular weight such as H₂O. In those cases, the dependence of λ on θ_i and γ_i is very weak.

We will define a linear absorbent as a material having the following two properties:

(1) The relation between the temperature and concentration in equilibrium with vapor at constant pressure is linear, of the form:

$$C = C_1 T + C_2 \quad (11)$$

and

(2) The heat of absorption is constant and independent of the temperature and concentration.

In terms of the dimensionless variables defined in equations (6c) and (10c), these two conditions become

$$\gamma_i = 1 - \theta_i \quad (11a)$$

and

$$\lambda = \text{const.} \quad (11b)$$

4. SOLUTION

Two different methods of solution were used to obtain the temperature and concentration distributions in the film: an analytical and a numerical method. A linear absorbent was assumed in both cases, and the results of the two methods were in excellent agreement. The equations in effect are equations (7) and (8) with the boundary conditions (9a) and (9b) at the entrance plane and the wall, respectively. The condition (9c) at the interface becomes, for the case of a linear

absorbent,

$$\theta + \gamma = 1 \quad \text{and} \quad \frac{\partial \theta}{\partial \eta} = \lambda \frac{\partial \gamma}{\partial \eta} \quad \text{at} \quad \eta = 1. \quad (12)$$

4.1. Analytical solution

The approach to the solution is similar to the one employed in ref. [13] for the case with the uniform velocity profile. Using the Fourier method, we write a separation-of-variables solution for equations (7) and (8), in the form of two infinite series of eigenfunctions, as follows:

$$\theta = \sum_{n=1}^{\infty} A_n F_n(\eta) e^{-\alpha_n^2 \zeta}, \quad (13a)$$

$$\gamma = 1 - \sum_{n=1}^{\infty} B_n G_n(\eta) e^{-\beta_n^2 \zeta} \quad (13b)$$

where α_n and β_n are the eigenvalues corresponding to the eigenfunctions F_n and G_n , respectively. The boundary conditions (12), which must be satisfied at any ζ , indicate that for every n ,

$$\alpha_n = \beta_n.$$

Substituting equations (13a) and (13b) into equations (7) and (8), we obtain the following two equations for the eigenfunctions:

$$\frac{d^2 F_n}{d\eta^2} + \frac{3}{2}(2\eta - \eta^2)\alpha_n^2 F_n = 0, \quad (14)$$

$$\frac{d^2 G_n}{d\eta^2} + \frac{3}{2}(2\eta - \eta^2)\frac{\alpha_n^2}{Le} G_n = 0, \quad (15)$$

with the boundary conditions at the wall, resulting from equations (9b):

$$G'_n(0) = 0 \quad (16a)$$

$$F'_n(0) = 0 \text{ for adiabatic wall, or } F(0) = 0 \text{ for constant temperature wall.} \quad (16b)$$

Another boundary condition to be satisfied by equations (14) and (15) is condition (12) at the interface, which yields

$$A_n F_n(1) = B_n G_n(1), \quad (17a)$$

$$A_n F'_n(1) = -\lambda B_n G'_n(1). \quad (17b)$$

We note that equations (17a) and (17b) for A_n and B_n are

homogeneous and have a solution only if the determinant equals zero, i.e. if

$$\frac{F'_n(1)}{F_n(1)} = -\lambda \frac{G'_n(1)}{G_n(1)}, \quad (18)$$

which is the condition for determining the eigenvalues α_n once a solution is obtained for F_n and G_n . The coefficients A_n and B_n can then be determined from the boundary condition (9a) by means of a Sturm–Liouville orthogonality condition at $\zeta = 0$.

A power series solution to equation (14) may be written in the form

$$F_n(\eta) = \sum_{i=0}^{\infty} a_{n,i} \eta^i \quad (19)$$

where, using the boundary condition (16b), we find

$$a_{n,0} = 1, \quad a_{n,1} = 0, \quad a_{n,2} = 0, \quad a_{n,3} = -\alpha_n^2/2 \quad \text{for adiabatic wall}$$

$$a_{n,0} = 0, \quad a_{n,1} = 1, \quad a_{n,2} = 0, \quad a_{n,3} = 0 \quad \text{for constant temperature wall} \quad (19a)$$

$$a_{n,i} = \frac{3}{2}\alpha_n^2(a_{n,i-4} - 2a_{n,i-3})/i(i-1) \quad \text{for } i \geq 4, \text{ both types of wall.}$$

Similarly, the solution to equation (15) may be written as

$$G_n(\eta) = \sum_{i=0}^{\infty} b_{n,i} \eta^i \quad (20)$$

where we find, with the aid of boundary condition (16a),

$$b_{n,0} = 1, \quad b_{n,1} = 0, \quad b_{n,2} = 0, \quad b_{n,3} = -\alpha_n^2/2Le$$

$$b_{n,i} = \frac{3}{2} \frac{\alpha_n^2}{Le} \frac{(b_{n,i-4} - 2b_{n,i-3})}{i(i-1)} \quad \text{for } i \geq 4. \quad (20a)$$

The eigenvalues are the roots of equation (18). An algorithm may therefore be employed where a guessed value of α_n is used in equations (19a) and (20a) to calculate the terms of the series $a_{n,i}$ and $b_{n,i}$, and, hence, the eigenfunctions $F_n(1)$ and $G_n(1)$. The results are then substituted into equation (18). If the latter is not satisfied, a different guess is taken until convergence is obtained.

Table 1 lists the first nine eigenvalues for a set of typical values of the parameters, $Le = 0.001$ and $\lambda = 0.01$. The table also shows the corresponding

Table 1. Eigenvalues and coefficients for typical values of the parameters: $\lambda = 0.01$, $Le = 0.001$

n	Adiabatic wall			Constant temperature wall		
	α_n	A_n	B_n	α_n	A_n	B_n
1	0	0.90909	0.90909	0.05765	0.026696	1.3373
2	0.10225	-0.14691	0.18984	0.16052	0.025699	-0.54334
3	0.19871	-0.12830	-0.19710	0.26269	0.026061	0.35597
4	0.29605	-0.10555	0.19098	0.36464	0.026789	-0.26809
5	0.39449	-0.08459	-0.17902	0.46640	0.027817	0.21567
6	0.49385	-0.06756	0.16558	0.56793	0.029115	-0.17983
7	0.59395	-0.05439	-0.15237	0.66916	0.030765	0.15338
8	0.69463	-0.04435	0.14006	0.76997	0.032680	-0.13206
9	0.79574	-0.03667	-0.12886	0.87079	0.035054	0.12027

coefficients A_n and B_n , which must be calculated to complete the solution. In order to do this, we have to formulate an orthogonality condition at $\zeta = 0$, which is of the Sturm–Liouville type, yet somewhat different from the standard form of the latter due to the coupled boundary condition (12).

Consider equation (14) for the eigenfunction F_n ; multiplying it by another eigenfunction F_m and integrating over the range of η yields

$$\begin{aligned} \frac{3}{2}\alpha_n^2 \int_0^1 (2\eta - \eta^2)F_m F_n \, d\eta &= - \int_0^1 F_m F_n'' \, d\eta \\ &= F_m(0)F_n'(0) - F_m(1)F_n'(1) + \int_0^1 F_m' F_n' \, d\eta, \end{aligned} \quad (21a)$$

similarly

$$\begin{aligned} \frac{3}{2}\alpha_m^2 \int_0^1 (2\eta - \eta^2)F_n F_m \, d\eta \\ &= F_n(0)F_m'(0) - F_n(1)F_m'(1) + \int_0^1 F_n' F_m' \, d\eta. \end{aligned} \quad (21b)$$

Subtracting equation (21b) from equation (21a) and using the boundary condition (16b), we obtain

$$\begin{aligned} \frac{3}{2}(\alpha_n^2 - \alpha_m^2) \int_0^1 (2\eta - \eta^2)F_n F_m \, d\eta \\ &= F_n(1)F_m'(1) - F_m(1)F_n'(1). \end{aligned} \quad (22a)$$

Repeating the same with equation (15) for the eigenfunction G_n , we find

$$\begin{aligned} \frac{3}{2Le} (\alpha_n^2 - \alpha_m^2) \int_0^1 (2\eta - \eta^2)G_n G_m \, d\eta \\ &= G_n(1)G_m'(1) - G_m(1)G_n'(1). \end{aligned} \quad (22b)$$

At this point we introduce the coupling between the equations, which is where the present orthogonality condition differs from the conventional. From equations (17a) and (17b)

$$\begin{aligned} F_n(1)F_m'(1) - F_m(1)F_n'(1) \\ &= -\lambda \frac{B_n B_m}{A_n A_m} [G_n(1)G_m'(1) - G_m(1)G_n'(1)] \end{aligned}$$

using this condition to combine equations (22a) and (22b) finally yields

$$\begin{aligned} (\alpha_n^2 - \alpha_m^2) \int_0^1 (2\eta - \eta^2) \\ \times (Le A_n A_m F_n F_m + \lambda B_n B_m G_n G_m) \, d\eta = 0 \end{aligned} \quad (23a)$$

which may be written as

$$\begin{aligned} \int_0^1 (2\eta - \eta^2)(Le A_n A_m F_n F_m \\ + \lambda B_n B_m G_n G_m) \, d\eta \quad \begin{cases} = 0 & \text{for } n \neq m, \\ \neq 0 & \text{for } n = m. \end{cases} \end{aligned} \quad (23b)$$

It should be noted that this type of ‘‘coupled’’ orthogonality condition had been developed and used

earlier in a problem involving sublimation in a duct [16].

We now return to the boundary condition (9a) and using equation (13) we find

$$\sum_{n=1}^j A_n F_n(\eta) = 0 \quad \text{and} \quad \sum_{n=1}^j B_n G_n(\eta) = 1 \quad (24)$$

therefore,

$$\begin{aligned} \sum_{n=1}^{\infty} \int_0^1 (2\eta - \eta^2)(Le A_n A_m F_n F_m \\ + \lambda B_n B_m G_n G_m) \, d\eta = \int_0^1 (2\eta - \eta^2)\lambda B_n G_m \, d\eta \end{aligned} \quad (25)$$

using the orthogonality condition (23b)

$$\begin{aligned} \int_0^1 (2\eta - \eta^2)(Le A_n^2 F_n^2 + \lambda B_n^2 G_n^2) \, d\eta \\ = \int_0^1 (2\eta - \eta^2)\lambda B_n G_n \, d\eta, \end{aligned} \quad (25a)$$

which provides one relation between A_n and B_n ; a second relation is available in either equation (17a) or (17b). Solving equations (25a) and (17a) for A_n and B_n yields

$$B_n = \frac{\lambda \int_0^1 (2\eta - \eta^2)G_n(\eta) \, d\eta}{\int_0^1 (2\eta - \eta^2) \left[Le \frac{G_n^2(1)}{F_n^2(1)} F_n^2(\eta) + \lambda G_n^2(\eta) \right] \, d\eta} \quad (26a)$$

$$A_n = B_n \frac{G_n(1)}{F_n(1)} \quad (26b)$$

The analytical solution is now complete. The algorithm mentioned earlier makes it possible to obtain the first few eigenvalues* from equation (18) without difficulty for most values of interest of the parameters. This is sufficient for an accurate calculation of θ and γ for moderate and large values of ζ due to the exponential terms in equation (13). For small values of ζ , however, a large number of eigenvalues is required. The recursive formulas (19a) and (20a) turn out to be unstable for large values of α_n , and it is increasingly difficult to obtain convergence of the series (19) and (20) for the eigenfunctions. An alternative method for obtaining the eigenvalues is through a numerical integration of equations (14) and (15). Rather than doing this, it was found more efficient to use a numerical method for solving the original equations (7) and (8) in their partial differential form, which will be described next. Yet, the analytical eigenvalue solution is very useful for a wide range of the parameters Le and λ , where enough eigenvalues can be calculated to cover a considerable range of ζ . For the small ζ 's, a similarity solution has been obtained similar to the one in ref. [14], which will also be described over.

* The sequence of the eigenvalues is such that a higher n corresponds to a larger value of α_n .

4.2. Numerical solution

The numerical technique used to solve the partial differential equations (7) and (8) was based on the so-called "method of lines" or "semi-discretization" [17]. The ζ - η plane of the film was divided into thin strips by means of lines parallel to the ζ axis. This discretization of the η -coordinate made it possible to express the second order derivative with respect to η in each of the equations in a finite-difference form. Thus, a first order ordinary differential equation, in ζ alone, was obtained along each line. Such an equation could be readily solved by means of an available ODE integrator using the boundary condition (9a). The integrator selects automatically the required step in ζ and varies it as necessary as the integration proceeds.

Some difficulty in applying this numerical method to the entire domain resulted from a singularity at the point $\zeta = 0, \eta = 1$. This is a singularity of the type often encountered in boundary layer problems, and is due to a discontinuity in the temperature and concentration between the interface and the entrance plane at this point. To overcome this problem, an analytical solution applicable close to the singular point was developed, which made it possible to calculate the values of the variables at some finite distance away from the point and begin the numerical solution from there. The analytical solution is similar to the one used in ref. [13] and will be described briefly here.

By defining a new variable

$$\eta_1 = 1 - \eta, \quad (27)$$

and recognizing that the term $(2\eta - \eta^2)$ is very close to unity near the singular point, we can rewrite equations (7) and (8) as

$$\frac{3}{2} \frac{\partial \theta}{\partial \zeta} = \frac{\partial^2 \theta}{\partial \eta_1^2}, \quad (28)$$

$$\frac{3}{2} \frac{\partial \gamma}{\partial \zeta} = Le \frac{\partial^2 \gamma}{\partial \eta_1^2}, \quad (29)$$

with the boundary conditions (9b) and (12) now applying at $\eta_1 \rightarrow \infty$ and $\eta_1 = 0$, respectively. It is then possible to find a similarity variable, combining both ζ and η_1 , for each of the equations and convert them from partial to ordinary ones. Using the common similarity technique, equation (28) becomes

$$\frac{d^2 \theta}{dz^2} = -2z \frac{d\theta}{dz} \quad (28a)$$

where $z = \eta_1 / (8\zeta/3)^{1/2}$. Equation (28a) may be integrated twice to give

$$\theta = k_1 \operatorname{erf}(z) + k_2 = k_1 \operatorname{erf}[\eta_1 / (8\zeta/3)^{1/2}] + k_2. \quad (28b)$$

In a similar manner we find from equation (29)

$$\gamma = k_3 \operatorname{erf}[\eta_1 / (8Le \zeta/3)^{1/2}] + k_4 \quad (29a)$$

where $k_1, k_2, k_3,$ and k_4 are constants of integration. Applying the boundary condition (9a) yields $k_1 = -k_2$ and $k_3 = -k_4$ since $\operatorname{erf}(\infty) = 1$. The boundary condition (9b) is satisfied automatically for both the

adiabatic and constant temperature walls. Then, applying the boundary condition (12) yields $k_1 + k_3 = -1$ and $k_1 = \lambda k_3 / (Le^{1/2})$ from which all the constants of integration can finally be determined. We thus obtain the following expressions for the dimensionless temperature and concentrations, in terms of the original variables

$$\theta = \frac{\lambda}{\lambda + Le^{1/2}} \left\{ 1 - \operatorname{erf} \left[\frac{3(1-\eta)^2}{8\zeta} \right]^{1/2} \right\}, \quad (30)$$

$$\gamma = \frac{Le^{1/2}}{\lambda + Le^{1/2}} \left\{ 1 - \operatorname{erf} \left[\frac{3(1-\eta)^2}{8Le \zeta} \right]^{1/2} \right\}, \quad (31)$$

which are valid for small ζ , for both the adiabatic and constant temperature wall cases. This is to be expected, since the effect of the wall cannot be felt until the boundary layer developing from the interface has had enough distance to fill the entire film thickness.

The similarity solution for small ζ has made it possible to use the numerical technique described earlier and overcome the problem associated with the discontinuity at the point $\zeta = 0, \eta = 1$. In addition, this solution is used to complement the eigenvalue solution whose usefulness at small ζ was limited by the number of obtainable eigenvalues.

5. RESULTS AND DISCUSSION

Figures 2 and 3 describe the general behavior of the temperature and concentration in the system as they vary with the normalized length ζ , for a typical set of values of the parameters Le and λ . Curves are given for θ and γ at the wall, (θ_w, γ_w) , the liquid bulk, $(\bar{\theta}, \bar{\gamma})$, and the liquid-vapor interface, (θ_i, γ_i) . The solid lines describe the results for the constant temperature wall and the broken lines for the adiabatic wall. This notation will be maintained for the rest of the curves in this section.

Initially, for very small ζ , the behavior is the same for the adiabatic and constant temperature wall cases. The liquid at the interface reaches thermodynamic equilibrium with the vapor immediately upon contact at $\zeta = 0$, but it takes some distance for the effect to diffuse into the film and be felt at the wall. Consequently, θ_w and γ_w remain essentially zero for small ζ while θ_i and γ_i remain almost constant at their initial values reached at $\zeta = 0$. These values are $\lambda / (\lambda + Le^{1/2})$ and $Le^{1/2} / (\lambda + Le^{1/2})$, respectively, as we find from the similarity solution for small ζ , equations (30) and (31).

As ζ increases for the adiabatic wall case, the wall, bulk, and interface temperatures increase monotonically toward a final common value and become closer and closer to each other. This steady increase is due to the fact that the heat of absorption is not being removed from the system. With the constant temperature wall, the interface temperature increases slightly, following the trend of small ζ , and the bulk temperature attempts to approach it as heat is conducted from the interface into the film. Then, both temperatures decrease toward zero as heat flows out of the system through the wall.

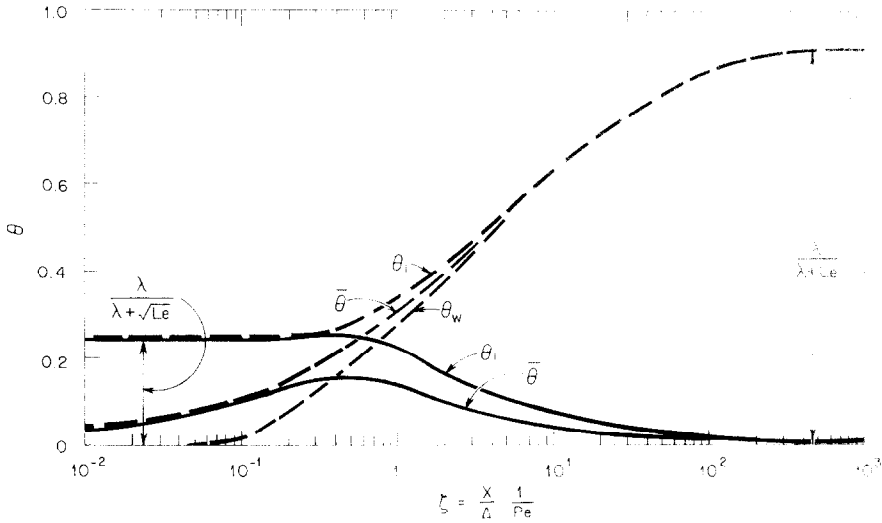


FIG. 2. Dimensionless wall, liquid bulk and interface temperatures as functions of the normalized length ζ for $Le = 0.001$ and $\lambda = 0.01$. Broken lines describe adiabatic wall; solid lines constant temperature wall.

The interfacial concentration in both cases follows a trend opposite to that of the interfacial temperature, since $\gamma_i = 1 - \theta_i$ [equation (12)]. The bulk concentration increases in both cases toward a final value equal to that of γ_i . It is interesting to note that in the adiabatic wall case $\bar{\gamma}$ increases with ζ while γ_i decreases. The asymptotic values of the dimensionless temperature and concentration may be found from the eigenvalue solution by substituting $\zeta \rightarrow \infty$ in equations (13a) and (13b). In the constant temperature wall case, the dimensionless temperature becomes equal to that of the wall ($\theta = 0$) and the concentration reaches the corresponding equilibrium value ($\gamma = 1$). In the adiabatic wall case the asymptotic temperature reflects some increase from the initial value [$\theta = \lambda/(Le + \lambda)$] and the corresponding equilibrium concentration

[$\gamma = Le/(\lambda + Le)$] is lower than the thermodynamically possible value of 1.

Figures 4 and 5 describe typical temperature and concentration profiles across the film for typical values of ζ . We notice that at small ζ the gradients of both quantities are very sharp and their variations are limited to a thin layer near the interface. As ζ increases, the effects at the interface diffuse toward the wall and the gradients become more moderate. There are, in fact, two boundary layers, one of temperature and one of concentration, which develop starting from the point of discontinuity ($\zeta = 0, \eta = 1$). The former develops considerably faster than the latter, as the thermal diffusivity α is larger than the mass diffusion coefficient D .

The effect of the heat of absorption λ is illustrated in Figs. 6 and 7 describing the temperature and

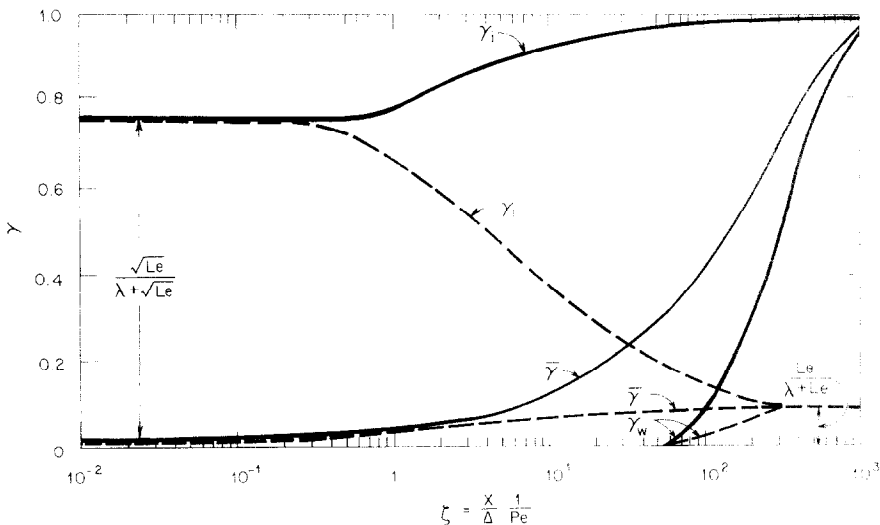


FIG. 3. Dimensionless wall, liquid bulk and interface concentrations as functions of the normalized length ζ for $Le = 0.001$ and $\lambda = 0.01$. Broken lines describe adiabatic wall; solid lines describe constant temperature wall.

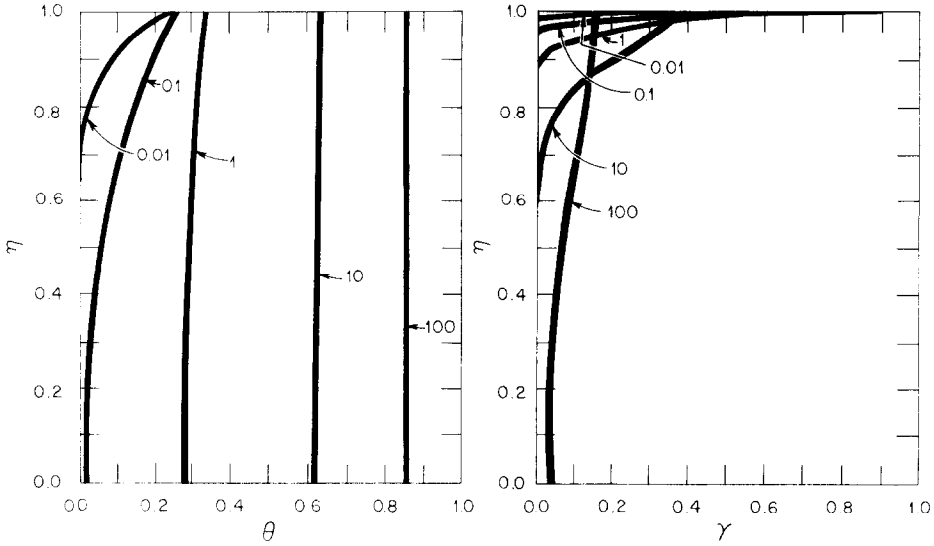


FIG. 4. Typical profiles of dimensionless temperature and concentration across the film at different values of ζ for adiabatic wall, $Le = 0.001$ and $\lambda = 0.01$.

concentration at the interface and at the wall, respectively. In the former, each curve represents either θ_i or γ_i when read on the corresponding scale, since $\theta_i + \gamma_i = 1$. We observe that the initial ($\zeta = 0$) value of the interface temperature increases with λ , and that of the interface concentration decreases, according to the formulas shown in Figs. 2 and 3. The same is true for the asymptotic values of temperature and concentration, respectively, both at the interface and at the wall. λ does not seem to have an effect on the distance required for the boundary layers to develop. Figure 7 indicates variations in wall temperature and concentration beginning approximately at the same value of ζ for all values of λ . At the limit of $\lambda = 0$ (negligible heat of absorption), θ is zero throughout the film, $\gamma_i = 1$ for all ζ

and γ_w varies as illustrated in Fig. 7. Under this condition, the results of our solution reduce to those of the models for isothermal mass transfer in a laminar falling film [5]. Also, in this case there is no difference between the adiabatic and constant temperature wall.

In addition to the results of the present solution for the adiabatic and constant temperature wall cases, Fig. 6 shows the results of the solution generated by Grigor'eva and Nakoryakov [13] with the assumption of a uniform velocity profile (for a constant wall temperature only). It is evident that the solution [13] shows the same initial and asymptotic behavior as the present one. However, the former underpredicts the length required to achieve a certain temperature or concentration level by about 40%.

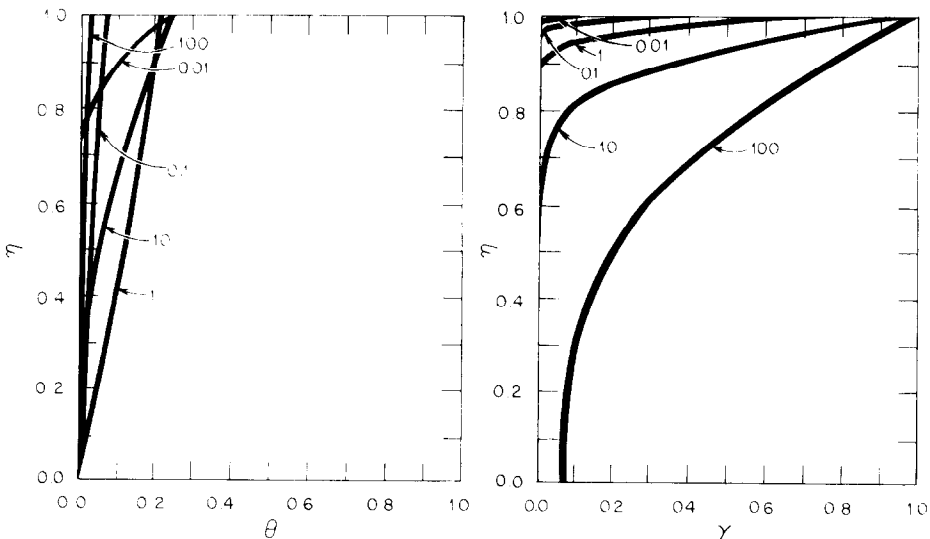


FIG. 5. Typical profiles of dimensionless temperature and concentration across the film at different values of ζ for constant temperature wall, $Le = 0.001$ and $\lambda = 0.01$.

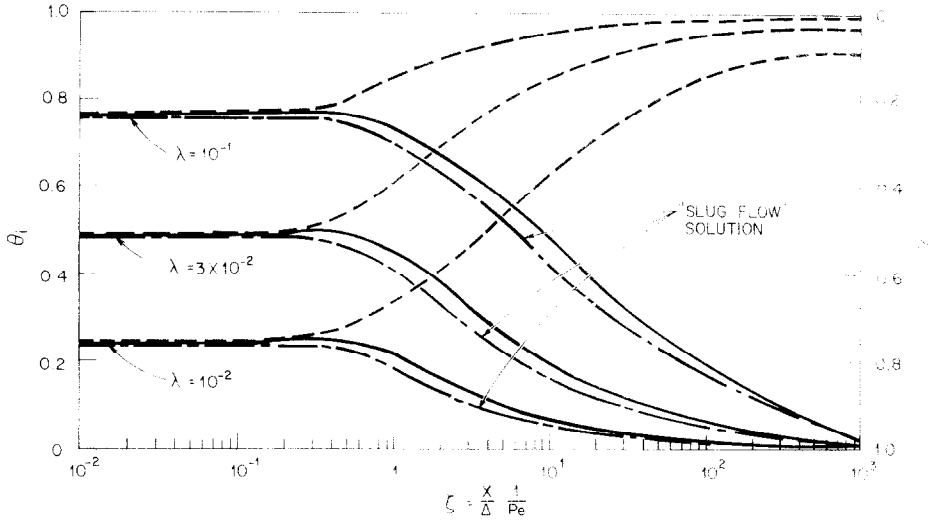


FIG. 6. Dimensionless interface temperature and concentration as functions of the normalized length ζ for $Le = 0.001$ and different values of λ . Broken lines describe adiabatic wall; solid lines describe constant temperature wall. The "slug flow" solution Grigor'eva and Nakoryakov [13] for the constant temperature wall is plotted for comparison.

The effect of the Lewis number is shown in Figs. 8 and 9 describing the temperature and concentration at the interface and at the wall, respectively. As in Fig. 6, each curve in Fig. 8 represents either θ_i or γ_i . The initial and asymptotic behavior are as predicted by the formulas given in Figs. 2 and 3. An increase in Le leads to a decrease in θ_i and to an increase in γ_i , at $\zeta = 0$ for both cases and at $\zeta \rightarrow \infty$ for the adiabatic wall case. In addition, Le has an effect on the development of the concentration boundary layer. The larger Le , the shorter the distance required for the concentration change to reach the wall, as shown by the curves for γ_w in Fig. 9.

Figure 10 shows the mass flux at the interface, μ_i , as a

function of ζ for $Le = 0.001$ and different values of λ . We observe that under all conditions, the rate of absorption is lower with an adiabatic wall than with a constant temperature wall. Initially, at small ζ , the curves for the two cases coincide. For larger ζ , after the thermal boundary layer has become fully developed, the heat removed at the wall enhances the mass transfer in the constant temperature wall case. The point at which the solid and broken curves part may serve as a measure for the length required for the full development of the thermal boundary layer. μ_i tends to zero at large ζ for both cases.

It is also observed that increasing λ reduces the mass flux, as expected. The curve for $\lambda = 0$ describes the case

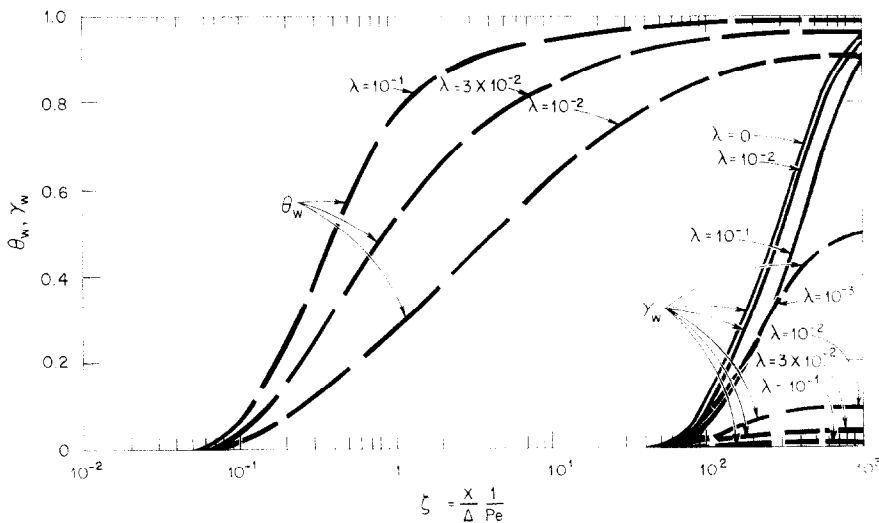


FIG. 7. Dimensionless wall temperature and concentration as functions of the normalized length ζ for $Le = 0.001$ and different values of λ . Broken lines describe adiabatic wall; solid lines describe constant temperature wall.

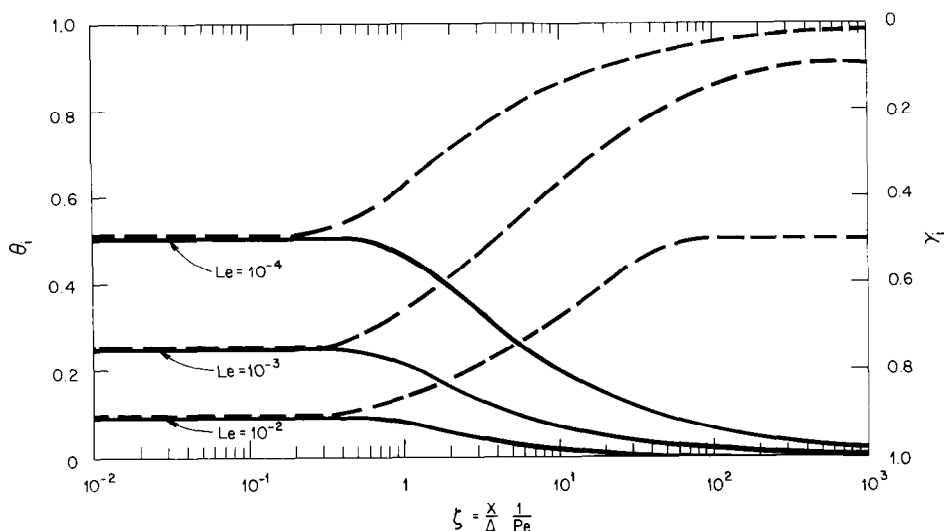


FIG. 8. Dimensionless interface temperature and concentration as functions of the normalized length ζ for $\lambda = 0.01$ and different values of Le . Broken lines describe adiabatic wall; solid lines describe constant temperature wall.

of isothermal mass transfer, in which μ_i is the largest possible for the given Le , and where there is no difference between the adiabatic and constant temperature wall.

6. HEAT AND MASS TRANSFER COEFFICIENTS

The literature is often somewhat ambiguous with regard to the definition of heat and mass transfer coefficients. This is particularly so in problems of simultaneous heat and mass transfer due to the coupling between the two processes. Yih and Seagrave [11] have used two different definitions of the Sherwood number, one based on $(C_1 - \bar{C})$ and the other

on $(C_1 - C_0)$. Nakoryakov and Grigor'eva [14] have defined it based on $(C_0 - C_e)$. Tamir and Taitel [5] have used an additional definition of an average Sherwood (or Nusselt) number based on a logarithmic mean concentration (or temperature) difference.

We will define the transfer coefficients based on the quantity difference which constitute the driving force for the transfer phenomenon. The coefficient of local mass transfer from the interface to the bulk of the liquid is defined through the Sherwood number as

$$Sh = \frac{h_M \Delta}{D} = \frac{\mu_i}{(\gamma_i - \bar{\gamma})} \tag{32}$$

The coefficient of local heat transfer from the interface

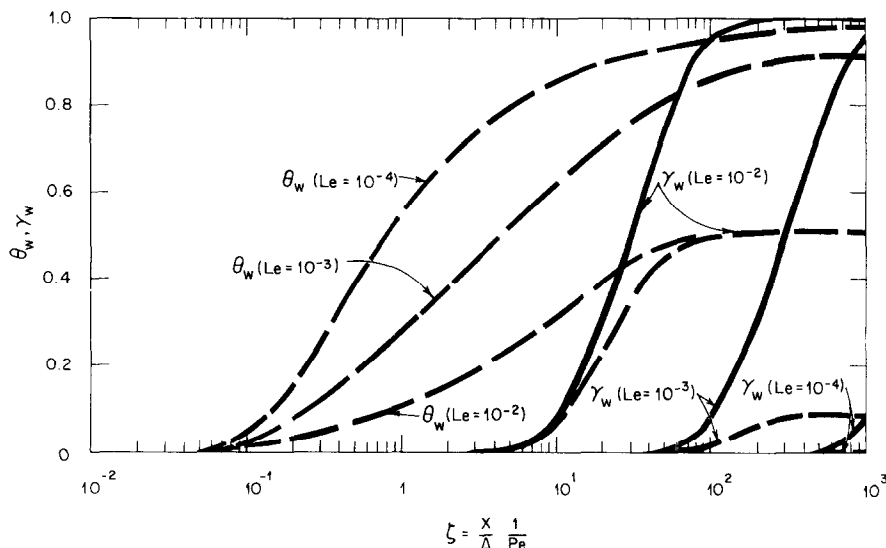


FIG. 9. Dimensionless wall temperature and concentration as functions of the normalized length ζ for $\lambda = 0.01$ and different values of Le . Broken lines describe adiabatic wall; solid lines describe constant temperature wall.

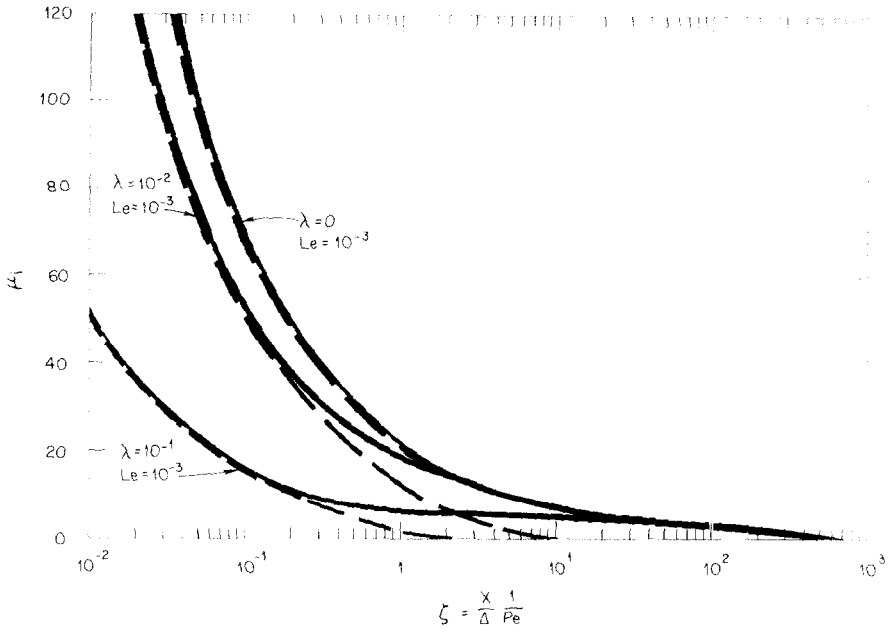


FIG. 10. Dimensionless mass flux at the interface as a function of the normalized length ζ for $Le = 0.001$ and different values of λ . Broken lines describe adiabatic wall; solid lines describe constant temperature wall.

to the bulk of the liquid is defined through the Nusselt number as

$$Nu = \frac{h_T \Delta}{k} = \frac{\mu_i \lambda}{(\theta_i - \bar{\theta})} \quad (33)$$

In the constant temperature wall case there is also a need to consider the heat transfer coefficient from the bulk of the fluid to the wall. Hence,

$$Nu' = \frac{h'_T \Delta}{k} = \frac{q_w}{\bar{\theta}} \quad (34)$$

Figure 11 describes the Sherwood number as a function of the normalized length ζ for different values of Le and λ . Sh is very large for small ζ and decreases toward an asymptotic value as ζ increases. We note, first, that for each set of conditions, Sh is greater with a constant temperature wall than with an adiabatic wall. The reasons for this are the same as those discussed in relation to μ_i (Fig. 10). The behavior in the two cases is the same for small ζ , and the discrepancy begins when the thermal boundary layer has reached the wall, increasing with ζ .

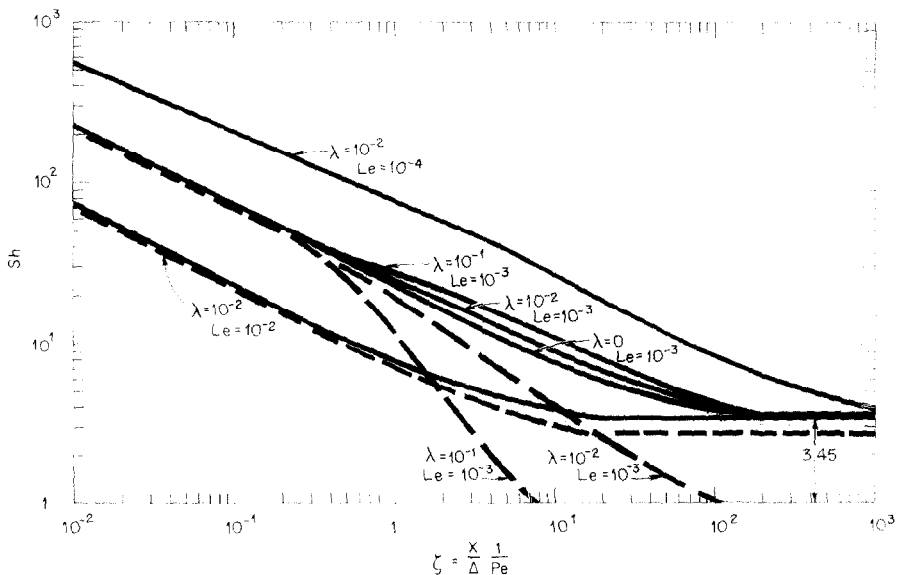


FIG. 11. Local Sherwood number as a function of the normalized length ζ for different values of Le and λ . Broken lines describe adiabatic wall; solid lines describe constant temperature wall.

In the case of a constant temperature wall, the effect of λ on Sh is small. For fixed λ , Sh is larger for smaller Le , contrary to what may be expected. This is so due to the fact that while the mass flux μ_1 increases with Le , the driving force $(\gamma_1 - \bar{\gamma})$ increases even faster. A smaller Lewis number requires a larger distance for the concentration boundary layer to become fully developed. For all combinations of λ and Le , the Sherwood number for a constant temperature wall tends to an asymptotic value of 3.45.

In the case of the adiabatic wall, increasing λ reduces Sh significantly for all values of Le . A larger λ leads to a greater deviation from the constant temperature wall behavior, this deviation shrinking to zero for $\lambda = 0$. For fixed λ , a larger Lewis number results in a smaller deviation. Unlike with the constant temperature wall, the asymptotic value of Sh depends on λ and Le , decreasing with the former and increasing with the latter.

The variations of Nu and Nu' with ζ are illustrated in Fig. 12, and are considerably less marked than that of Sh . In the initial region of development of the thermal boundary layer, Nu decreases in the same manner for the adiabatic and for the constant temperature wall cases. In this region, Nu' is zero, as the effects at the interface have not reached the wall. Beyond that region there is little variation in Nu , which tends to the asymptotic values of 4.23 and 2.65 for the adiabatic and constant temperature wall, respectively. Nu' reaches an asymptotic value of 1.60. The above behavior is practically unaffected by λ and Le for a wide range of values of these parameters.

The results of the "slug flow" model by Grigor'eva and Nakoryakov [13] show the same general behavior, but the actual values of the coefficients deviate by about

20% from those of the present analysis. With the assumption of a uniform velocity profile and a constant temperature wall, the asymptotic value of Sh is 3.00, and that of both Nu and Nu' is 2.00.

7. CONCLUSIONS

A model was developed for analysis of the combined heat and mass transfer processes in absorption of vapor in laminar liquid films. The energy and diffusion equations were solved with an equilibrium boundary condition at the vapor-liquid interface. Two cases of practical importance were considered—a constant temperature wall and an adiabatic wall. Two methods of solution (analytical and numerical) were used with very good agreement between their respective results. The solution was carried out for a linear absorbent—a mixture with a linear temperature-concentration equilibrium relation and a constant heat of absorption. The techniques of solution are suitable, however, for non-linear absorbents with given characteristics.

The results of the solution describe the development of the thermal and concentration boundary layers and the variation of the temperatures, concentrations, and heat and mass fluxes. These quantities in their normalized, dimensionless forms depend on two characteristic parameters of the system: the Lewis number Le and the dimensionless heat of absorption λ . The length in the direction of flow is normalized with respect to the Peclet number and the film thickness. In the constant temperature wall case, the dimensionless temperature and concentration reach asymptotic values of 0 and 1, respectively. In the adiabatic wall case the asymptotic temperature and concentration are $\lambda/(\lambda + Le)$ and $Le/(\lambda + Le)$, respectively.

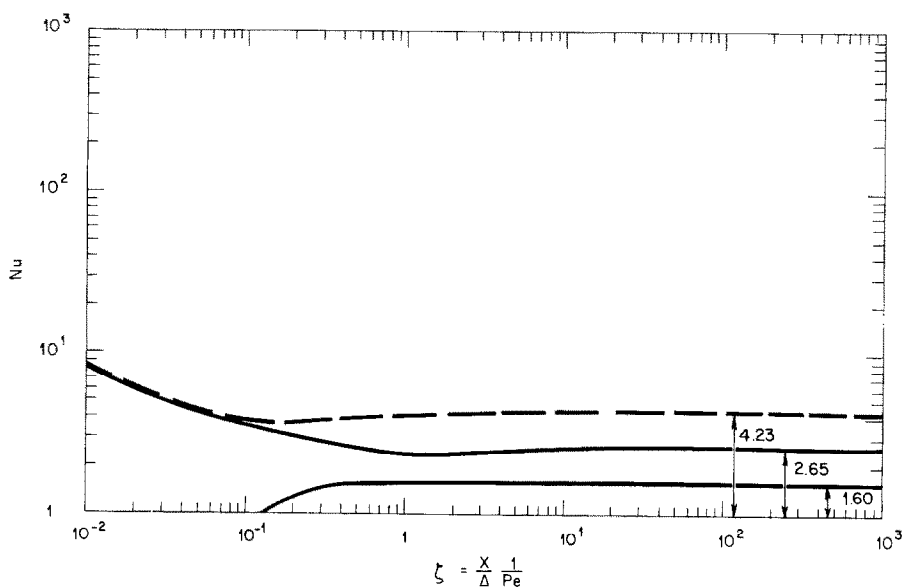


FIG. 12. Local Nusselt number as a function of the normalized length ζ . The curves are practically unaffected by variation in Le between 10^{-3} and 10^{-2} , and by variations in λ between 10^{-3} and 10^{-1} . Broken lines describe adiabatic wall; solid lines describe constant temperature wall.

Heat and mass transfer coefficients for the system were calculated. The Sherwood number for mass transfer from the vapor-liquid interface to the bulk of the film reaches an asymptotic value of 3.45, with fully developed boundary layers for the constant temperature wall. Lower values are obtained with an adiabatic wall. The Nusselt number for heat transfer from the interface to the bulk reaches under the same conditions values of 4.23 and 2.65 for the adiabatic and constant temperature wall, respectively. The Nusselt number for heat transfer from the bulk to the wall reaches 1.60.

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TRANSMISSION SIMULTANÉE DE LA CHALEUR ET DE LA MASSE AU COURS DE LEUR ABSORPTION PAR UNE MINCE COUCHE LIQUIDE DONT L'ÉCOULEMENT EST LAMINAIRE

Résumé—Cet article présente une analyse théorique du processus de transmission combinée de chaleur et de masse ayant lieu lors de l'absorption d'un gaz ou d'une vapeur dans une mince couche liquide laminaire. Les équations d'énergie et de diffusion sont résolues simultanément pour déterminer les variations de température et de concentration au point de l'interface gaz-liquide et au contact de la surface sous-jacente. Deux cas particulièrement intéressants sont traités: une condition isotherme et une cloison adiabatique. On en conclut que les chiffres de Nusselt et Sherwood dépendent des chiffres de Peclet et Lewis aussi bien que des propriétés d'équilibre des matières employées.

GLEICHZEITIGER WÄRMEÜBERGANG UND STOFFAUSTAUSCH BEI FILMABSORPTION MIT LAMINARER STRÖMUNG

Zusammenfassung—Die vorliegende Arbeit beschreibt eine theoretische Analyse des Wärmeübergang-Stoffaustauschvorgangs, der bei der Absorption eines Gases oder Dampfes in einen laminaren flüssigen Film stattfindet. Die Energie- und Diffusionsgleichungen werden gleichzeitig aufgelöst, damit die Temperatur- und Konzentrationsänderungen an der Grenzfläche zwischen Flüssigkeit und Gas und an der Wand ermittelt werden können. Zwei interessierende Fälle werden betrachtet: eine konstante Temperatur und eine adiabatische Wand. Es wird festgestellt, daß die Nusselt- und Sherwood-Zahlen von den Peclet- und Lewis-Zahlen sowie von den Gleichgewichtskennzahlen der Arbeitsstoffe abhängen.

**СОВМЕСТНОЕ РАССМОТРЕНИЕ ТЕПЛОТДАЧИ И ПЕРЕНОСА ВЕЩЕСТВ
ПРИ ПЛЕНОЧНОМ ПОГЛОЩЕНИИ С ЛАМИНАРНЫМ ПОТОКОМ**

Аннотация— Настоящая работа представляет собой теоретический анализ совместных процессов теплоотдачи и переноса веществ при поглощении газа или паров в ламинарную жидкую пленку. Уравнения энергии и диффузии решаются совместно, в результате чего получаются величины колебания температуры и концентрации у поверхности раздела жидкой и газовой фаз и у стенки. Рассматриваются два случая—постоянной температуры и адиабатической стенки. Оказывается что Нуссельта число и Шервуда число зависят как от числа Пекле и числа Льюиса, так и от равновесных свойств рабочих веществ.